STUDY OF QUEUING THEORY AND ITS APPLICATION IN OPERATION RESEARCH

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***Abstract*—In this present paper, we studied a queuing theory. The purpose of this paper is to seen that as a system gets congested, the service delay in the system increases. A good understanding of the relationship between congestion and delay is essential for designing effective congestion control algorithms. Queuing Theory provides all the tools needed for this analysis [1-9].**

***Index Terms*—Queuing Theory, Operation Research, Little's Theorem.**

I. Introduction

Queuing Theory provides all the tools needed for this analysis [1].

Communication Delays

Let us understand the different components of delay in a messaging system. The total delay experienced by messages can be classified into the following categories:

Processing Delay

* This is the delay between the times of receipt of a packet for transmission to the point of putting it into the transmission queue.
* On the receive end, it is the delay between the time of reception
of a packet in the receive queue to the point of actual processing of the message.
* This delay depends on the CPU speed and CPU load in the system.

Queuing Delay

* This is the delay between the point of entry of a packet in the transmit queue to the actual point of transmission of the message.
* This delay depends on the load on the communication link.

Transmission Delay

* This is the delay between the transmission of first bit of the packet to the transmission of the last bit.
* This delay depends on the speed of the communication link.

Propagation Delay

* This is the delay between the point of transmission of the last bit of the packet to the point of reception of last bit of the packet at the other end.
* This delay depends on the physical characteristics of the
communication link.

Retransmission Delay

* This is the delay that results when a packet is lost and has to be retransmitted.
* This delay depends on the error rate on the link and the protocol used for retransmissions.

Little's Theorem

The average number of customers (N) can be determined from the following equation:

$$N = \lambda T$$

Here lambda is the average customer arrival rate and T is the average service time for a customer.
Proof of this theorem can be obtained from any standard textbook on queueing theory. Consider the example of a restaurant where the customer arrival rate (lambda) doubles but the customers still spend the same amount of time in the restaurant (T). This will double the number of customers in the restaurant (N). By the same logic if the customer arrival rate remains the same but the customers service time doubles, this will also double the total number of customers in the restaurant [2].

Waiting in line for service is one of the most unpleasant experiences of life on this world. Barrers (1957) says, in certain queuing processes a potential customer is considered “lost” if the system is busy at the time service is demanded. If not served during this time, the customer leaves the system and is considered lost. In queuing system the customer satisfaction can be increased by constructing control charts for N and providing control limits for N so as to make effective utilization of time. If control limits are displayed so that customer can have prior idea about control limits. For any queuing system/model, average queue length and average waiting time are the main observable characteristics. Customer wants to have the waiting time in the system as minimum as possible that is queue length should be small. To monitor the average queue length of the system through control charts, In this paper Control limits are established.

Construction of Control chart for random queue length N is done using method suggested by Haim Shore and also by using traditional Shewhart method. The performance of these two control charts is compared using performance measure ARL. It is observed that using these control limits, the performance of the system may be improved [3].

With this factor, an attempt is made to find control limits for random queue length N for Power Supply Model (M/M/∞) :(∞/FIFO) queuing model. The pioneering work in this direction was made by Haim Shore in 2000. Two control chart C_1 and C_2 are constructed for random queue length N for (M/M/1):(∞/FCFS) queuing model by Khaparade and Dhabe (2010) and also C_3 and C_4 are constructed for random queue length N for (M/M/1):(∞/FCFS) queuing model by Khparde and Dhabe(2011) where

Control Chart C_1: is the Shewhart control chart
Control Chart C_2: is the control chart using method of Weighted variance
Control Chart C_3: is control chart for random queue length N based on skewness and
Control Chart C_4: is control chart using Nelson’s Power transformation

In this paper construction of control charts C_3 for random queue length N for “power supply model” queuing model is done using method based on skewness. Performance of control chart C_3 is compared with Control chart C_4.

II. GENERALIZED MODEL: BIRTH AND DEATH PROCESS

This model deals with a queuing system having single service channel, Poisson input with no limit on system capacity. Arrivals can be considered as birth to the system, whereas a departure can be looked upon as a death [4].

n = number of customers in the system
λ_n = arrival rate of customers given to n customersin the system,
μ_n = departure rate of customers given n customers in the system, and
P_n = steady-state probability of n customers in the system

The model determines the values of P_n in terms of λ_n and μ_n. Now we use the following Axioms of Poisson Process:

Axiom 1:
The number of arrivals is non-overlapping intervals are
statistically independent, that is, the process has independent increments.

Axiom 2:
The probability of more than one arrival occurs between time t and time t + Δt is (t); that is the probability of two or more arrivals during the small-time interval Δt is negligible.
Thus,

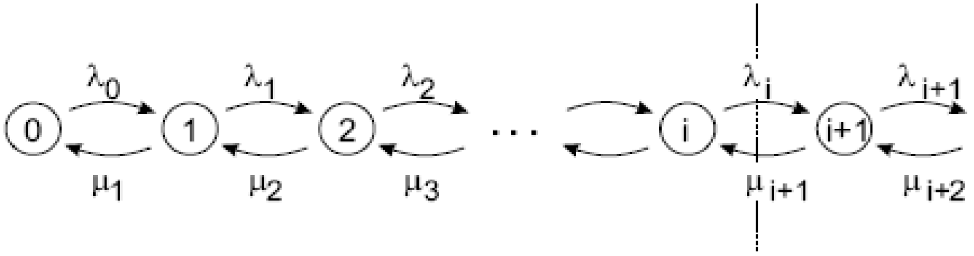
$$P_0(t) + P_1(\Delta t) + o(\Delta t) = 1$$

Axiom 3:
The probability that an arrival occurs between time t and time t + Δt is equal to λΔt +0(Δt)
Thus, $P_1(\Delta t) = \lambda\Delta t + o(\Delta t)$
Where λ is constant and is independent of the total number of arrivals up to time t, Δt is an increment element, and 0(Δt) represents the terms such that

$$\lim_{\Delta t \to \infty} \left(\frac{o(\Delta t)}{\Delta t} \right) = 0.$$

From the axioms we observed that an arrival during the small-time interval Δt is negligible. This implies that for > 0, state n can change only two possible states: states n-1 when a departure occurs at the rate μ_n and states n+1 when an arrival occurs at rate λ_n. State 0 can only change to state I when an arrival occurs at the rate λ_0. Since no departure is possible when the system is empty, μ_0 is undefined [5].

Under steady-state conditions, for > 0 , the rate of flow into and out of state nn must be equal. This is illustrated in the transition-rate diagram given below:



The balance equation is:
Expected rate of flow into state 6 = Expected rate of flow out of state n
i.e. $\lambda_{n-1}P_{n-1} + \mu_{n+1}P_{n+1} = \lambda_n P_n + \mu_n P_n$
and $\mu_1 P_1 = \lambda_0 P_0$
using the iterative procedure, we have

$$P_1 = \frac{\lambda_0}{\mu_1} P_0$$

$$P_2 = \frac{\lambda_0 + \mu_1}{\mu_2} P_1 - \frac{\lambda_0}{\mu_2} P_0$$

$$= \frac{\lambda_0 \lambda_1}{\mu_2 \mu_1} P_0$$

$$P_3 = \frac{\lambda_2 + \mu_2}{\mu_3} P_2 - \frac{\lambda_1}{\mu_3} P_0$$

$$= \frac{\lambda_0 \lambda_1 \lambda_2}{\mu_1 \mu_2 \mu_3} P_0$$

In general, we can write the following formula

$$P_n = \frac{\lambda_{n-1}\lambda_{n-2}\lambda_{n-3}\cdots\cdots\lambda_0}{\mu_n \mu_{n-1}\mu_{n-2}\cdots\cdots\mu_1} P_0 \quad \text{or} \quad P_n = P_0 + \prod_{i=1}^{n} \frac{\lambda_i}{\mu_{i+1}}$$

Now,

$$P_{n+1} = \frac{\lambda_n + \mu_n}{\mu_{n+1}} P_{n+1} - \frac{\lambda_{n-1}}{\mu_{n+1}} P_{n-1}$$

$$P_{n+1} = P_0 \prod_{i=1}^{n} \frac{\lambda_i}{\mu_{i+1}}$$

Thus, by mathematical induction the general value of P_n holds for all. To obtain the value of P_0, we use the boundary condition

$$\sum_{n=0}^{\infty} P_n = 1 \ and \ P_0 + \sum_{n=1}^{\infty} P_n = 1$$

$$P_0 = \left(1 + \sum_{i=1}^{\infty} \prod_{i=1}^{n-1} \frac{\lambda_i}{\mu_{i+1}}\right)^{-1}$$

III. POWER SUPPLY MODEL

This model describes a situation where an electric circuit supplies power to 'a' customers [6]. The requirements of customers are assumed to follow Poisson distribution with parameter λ, and the supply schedule also follows Poisson distribution with parameter μ. Using the balance equation for the generalized model (Birth and Death Process) discussed above with

$$\lambda_n = (a - n)\lambda \qquad \text{and} \qquad \mu_n = n\mu$$

We have,

$$P_n = \frac{1}{n!} a(a-1) \ldots \ldots \ldots \ldots (a - n + 1) \left(\frac{\lambda}{\mu}\right)^n P_0$$

And,

$$P_a = \frac{a!}{a!} \left({}^{\lambda}/_{\mu} \right)^a P_0$$

Using the boundary condition

$$\sum_{n=0}^{\infty} P_n = 1$$

We get,

$$P_0\left[1 + {}^{\lambda}/_{\mu} + {}^{a(a-1)}/_{2!}\left({}^{\lambda}/_{\mu}\right)^2 + \cdots + \left({}^{\lambda}/_{\mu}\right)^a\right] = 1$$

i.e.,

$$P_0\left[1 + {}^{\lambda}/_{\mu}\right]^a = 1 \qquad \text{or} \qquad P_0 = \left[\frac{\mu}{\mu+\lambda}\right]^a$$

Therefore,

$$P_n = \frac{1}{n!}a(a-1)\ldots\ldots\ldots\ldots(a-n+1)\left({}^{\lambda}/_{\mu}\right)^n\left(\frac{\mu}{\mu+\lambda}\right)^a$$

$$= \frac{a!}{(a-n)!n!}\left({}^{\lambda}/_{\mu}\right)^n\left(\frac{\mu}{\mu+\lambda}\right)^a$$

$$P_n = \binom{a}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^a\left(\frac{\mu}{\mu+\lambda}\right)^{a-n} \qquad (1)$$

which is a binomial distribution.

Let random variable W_S denote the waiting time spent in the system by the customer. This includes both the waiting time and service time. The p.d.f. of random variable W_S is given by:

$$f(W_S) = (\mu-\lambda)e^{-(\mu-\lambda)W_S} \qquad ,W_S > 0 \qquad (2)$$

Thus, W_S follows an exponential distribution with mean

$$E(W_S) = \int_0^{\infty} W_S f(W_S)dW_S$$

$$= \int_0^{\infty} W_S\,(\mu-\lambda)e^{-(\mu-\lambda)W_S}dW_S$$

$$E(W_S) = \frac{1}{\mu-\lambda} \qquad (3)$$

$$E(W_S^2) = \frac{2}{(\mu-\lambda)^2} \qquad (4)$$

Using (3) and (4), The variance is given by

$$V(W_S) = E(W_S^2) - [E(W_S)]^2$$

$$V(W_S) = \frac{1}{(\mu-\lambda)^2} \qquad (5)$$

The distribution function of W_S is

$$F(x) = P(W_S \le x)$$

$$= \int_0^x f(W_S)dW_S$$

$$= \int_0^x (\mu - \lambda)e^{-(\mu-\lambda)W_S}dW_S$$

$$= 1 - e^{-(\mu-\lambda)x}, x > 0$$

Hence,

$$F(x) = 1 - e^{-(\mu-\lambda)x}, x > 0$$

The probability distribution of N is a geometric distribution with parameter (1 − □). The steady state distribution of the random variable N depends on two parameters μ and λ only through their ratio [7]. Using equation (1), the average number of customers and variance of queue length in the system is given by

$$E(N) = \mu_1'$$

$$= \sum_{a=1}^{n} n.P_n$$

$$= \sum_{a=1}^{n} n.\binom{a}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^{a}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$$

$$= \sum_{a=1}^{n} n\left(\frac{a(a-1)!}{n(n-1)!}\right)\left(\frac{\lambda}{\mu+\lambda}\right)^{a-1}\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$$

$$= \left(\frac{\lambda}{\mu+\lambda}\right)a\sum_{a=1}^{n}\binom{a-1}{n-1}\left(\frac{\lambda}{\mu+\lambda}\right)^{a-1}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$$

Using the boundary condition
We get

$$\mu_1' = \left(\frac{\lambda}{\mu+\lambda}\right)a \qquad (6)$$

$$E(N^2) = \mu_2'$$

$$E(N^2) = E[N(N-1) + N]$$

$$= \sum_{a=1}^{n}[n(n-1)]\binom{a}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^{a}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n} + \sum_{a=1}^{n} n.\binom{a}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^{a}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$$

$$= \sum_{a=1}^{n} n.(n-1)\left(\frac{a(a-1)(a-2)!}{n(n-1)(n-2)!\,(a-n)!}\right)\left(\frac{\lambda}{\mu+\lambda}\right)^{a-2}\left(\frac{\lambda}{\mu+\lambda}\right)^{2}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$$

$$+ \sum_{a=1}^{n} n\left(\frac{a(a-1)!}{n(n-1)!}\right)\left(\frac{\lambda}{\mu+\lambda}\right)^{a-1}\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$$

$$= \left(\frac{\lambda}{\mu+\lambda}\right)^{2} a(a-1)\sum_{a=1}^{n}\binom{a-2}{n-2}\left(\frac{\lambda}{\mu+\lambda}\right)^{a-2}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$$

$$+ \left(\frac{\lambda}{\mu+\lambda}\right)a\sum_{a=1}^{n}\binom{a-1}{n-1}\left(\frac{\lambda}{\mu+\lambda}\right)^{a-1}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$$

By using the boundary condition, we get

$$\mu_2' = \left(\frac{\lambda}{\mu+\lambda}\right)^2 a(a-1) + \left(\frac{\lambda}{\mu+\lambda}\right) a$$

$$= a\left(\frac{\lambda}{\mu+\lambda}\right)\left[1 + (a-1)\left(\frac{\lambda}{\mu+\lambda}\right)\right] \tag{7}$$

Using (6) and (7), the variance of queue length in the system is

$$V(N) = E(N^2) - [E(N)^2]$$

$$\mathrm{V(N)} =$$

$$\left[\sum_{a=1}^{n}[n(n-1)]\binom{a}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^{a}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n} + \sum_{a=1}^{n} n.\binom{a}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^{a}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}\right]$$

$$-\left[\sum_{a=1}^{n} n.\binom{a}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^{a}\left(\frac{\mu}{\mu+\lambda}\right)^{a-n}\right]^2$$

Hence,

$$V(N) = \left[a\left(\frac{\lambda}{\mu+\lambda}\right)\left[1 + (a-1)\left(\frac{\lambda}{\mu+\lambda}\right)\right]\right] - \left[\left(\frac{\lambda}{\mu+\lambda}\right)a\right]^2$$

$$V(N) = a\left(\frac{\lambda}{\mu+\lambda}\right) + \mathrm{a}(\mathrm{a}-1)\left(\frac{\lambda}{\mu+\lambda}\right)^2 - \mathrm{a}^2\left(\frac{\lambda}{\mu+\lambda}\right)^2$$

$$V(n) = a\left(\frac{\lambda}{\mu+\lambda}\right)\left(1 - \left(\frac{\lambda}{\mu+\lambda}\right)\right)$$

$$V(N) = a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right) \tag{8}$$

Using (8) The standard deviation is calculated as

$$\sigma = \sqrt{V(N)}$$

So,

$$\sigma = \sqrt{\left[a\left(\frac{\lambda}{\mu+\lambda}\right)\left[1 + (a-1)\left(\frac{\lambda}{\mu+\lambda}\right)\right]\right] - \left[\left(\frac{\lambda}{\mu+\lambda}\right)a\right]^2}$$

$$\sigma = \sqrt{a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)} \tag{9}$$

Now,

$$E(N^3) = \mu_3'$$

For this we use

$$N^3 = N(N-1)(N-2) + 3N^2 - 2N$$

We have,

$$E(N^3) = E[N(N-1)(N-2)] + 3E(N^2) - 2\,E(N)$$

$$= a(a-1)(a-2)\left(\frac{\lambda}{\mu+\lambda}\right)^3$$

$$+ 3\left[a\left(\frac{\lambda}{\mu+\lambda}\right)\left[1 + (a-1)\left(\frac{\lambda}{\mu+\lambda}\right)\right] - 2\left[\left(\frac{\lambda}{\mu+\lambda}\right)a\right]\right]$$

$$E(N^3) = a(a-1)(a-2)\left(\frac{\lambda}{\mu+\lambda}\right)^3 + 3a(a-1)\left(\frac{\lambda}{\mu+\lambda}\right)^2 + a\left(\frac{\lambda}{\mu+\lambda}\right) \tag{10}$$

Hence, we get,

$$E(N^3) = \mu_3' = a(a-1)(a-2)\left(\frac{\lambda}{\mu+\lambda}\right)^3 + 3a(a-1)\left(\frac{\lambda}{\mu+\lambda}\right)^2 + a\left(\frac{\lambda}{\mu+\lambda}\right) \tag{11}$$

Using the (6), (7) and (11) the third central moment is given by

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2[\mu_1']^3$$

$$\begin{aligned}\mu_3 = \Bigg[\Bigg(& a(a-1)(a-2)\left(\frac{\lambda}{\mu+\lambda}\right)^3 + 3a(a-1)\left(\frac{\lambda}{\mu+\lambda}\right)^2 + a\left(\frac{\lambda}{\mu+\lambda}\right) \\ & - 3\left(a\left(\frac{\lambda}{\mu+\lambda}\right)\left[1 + (a-1)\left(\frac{\lambda}{\mu+\lambda}\right)\right]\right)\left(\left(\frac{\lambda}{\mu+\lambda}\right)a\right) \\ & + 2\left(\left(\frac{\lambda}{\mu+\lambda}\right)a\right)^3\Bigg)\Bigg]\end{aligned}$$

$$\begin{aligned}\mu_3 = a\left(\frac{\lambda}{\mu+\lambda}\right)\Bigg[& -3a\left(\frac{\lambda}{\mu+\lambda}\right)^2 + 3a\left(\frac{\lambda}{\mu+\lambda}\right) + 2\left(\frac{\lambda}{\mu+\lambda}\right)^2 - 3\left(\frac{\lambda}{\mu+\lambda}\right) + 1 \\ & - 3a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)\Bigg]\end{aligned}$$

$$\mu_3 = a\left(\tfrac{\lambda}{\mu+\lambda}\right)\left[3a\left(\tfrac{\lambda}{\mu+\lambda}\right)(1 - \left(\tfrac{\lambda}{\mu+\lambda}\right) + 2\left(\tfrac{\lambda}{\mu+\lambda}\right)^2 - 3\left(\tfrac{\lambda}{\mu+\lambda}\right) + 1 - 3\mathrm{a}\left(\tfrac{\lambda}{\mu+\lambda}\right)\left(\tfrac{\mu}{\mu+\lambda}\right)\right]$$

$$\mu_3 = a\left(\frac{\lambda}{\mu+\lambda}\right)\left[2\left(\frac{\lambda}{\mu+\lambda}\right)^2 - 3\left(\frac{\lambda}{\mu+\lambda}\right) + 1\right]$$

$$\mu_3 = a\left(\tfrac{\lambda}{\mu+\lambda}\right)\left[2\left(\tfrac{\lambda}{\mu+\lambda}\right)^2 - 2\left(\tfrac{\lambda}{\mu+\lambda}\right) + \left(\tfrac{\mu}{\mu+\lambda}\right)\right]$$

$$\mu_3 = a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)\left[1 - 2\left(\frac{\lambda}{\mu+\lambda}\right)\right]$$

$$\mu_3 = a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)\left[\left(\frac{\mu}{\mu+\lambda}\right) + \left(\frac{\lambda}{\mu+\lambda}\right) - 2\left(\frac{\lambda}{\mu+\lambda}\right)\right]$$

$$\mu_3 = a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)\left[\frac{\mu}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda}\right] \tag{12}$$

Let $S_k(N)$ denote the traditional measure of skewness for random queue length namely,

$$S_k(N) = \frac{\mu_3}{\sigma^3}$$

where μ_3 is the third central moment. Using (9) and (12)

$$S_k(N) = \frac{a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)\left[\frac{\mu}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda}\right]}{\left(\sqrt{a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)}\right)^3}$$

$$= \sqrt{\frac{a^2\left(\frac{\lambda}{\mu+\lambda}\right)^2\left(\frac{\mu}{\mu+\lambda}\right)^2\left(\frac{\mu}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda}\right)^2}{a^3\left(\frac{\lambda}{\mu+\lambda}\right)^3\left(\frac{\mu}{\mu+\lambda}\right)^3}}$$

$$= \left[\frac{\left[\frac{\mu}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda}\right]^2}{a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)}\right]^{1/2}$$

$$S_k(N) = \frac{\left[\frac{\mu}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda}\right]}{\sqrt{a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)}}$$

IV. SHEWHART'S CONTROL CHART C1

The basic principle underlying all the Shewhart type control charts is that the distribution of the plotted statistic may be approximated by a normal distribution, with parameters that preserve the true mean and standard deviation. The control charts have very widely spread applications which means that they are very useful and it also speaks about the general robustness of the Shewhart control chart to deviation from normality[8].

Traditional Shewhart chart ignores skewness of the plotted statistic. Some times the skewness is too large to be ignored. Ignoring skewness affects the performance of the control chart and resuls in high false alarm rate. Many such circumstances have been extensively studied and reported in literature. This has motivated the various control schemes that take account of non-normality of the monitoring statistic. Haim Shore[2000] have developed control chart for N for (M/M/s):(∞/FCFS) queuing models. He has developed a general framework for constructing Shewheart like control charts for attributes based on fitting a quantile function that preservsall first three moments of the plotted statistic.

The formulae for calculating the control limits are also based on these three moments. To make these control limits more accurate, the skewness measure used in the calculation is inflated by 44%. This inflation rate gives more accurate control limits for diversely shaped attribute distribution like binomial, the Poisson, the Geometric, the negative binomial.

Comprehensive numerical assessment of this new approach with regard to binomial, the Poisson, the Geometric, the negative binomial is carried out and it is shown that this new approach is very effective, convenient to use and most importantly there is no major departure from Shewhart control charts [9].

When it is possible to specify standard values for the process mean and standard deviation, we may use these standards to establish the control chart. Then the parameters of the chart are

$$\begin{aligned} UCL &= \mu + A\sigma \\ CL &= \mu \\ LCL &= \mu - A\sigma \end{aligned}$$

Where the quantity say, is a $A = \frac{3}{\sqrt{a}}$ constant that depends on a.
Using expressions (6) and (9), the control limits for 'Power Supply Model' are calculated as follows:

$$UCL = \left(\frac{\lambda}{\mu+\lambda}\right)a + A\sqrt{a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)}$$

$$UCL = \frac{\rho}{(1+\rho)} + A\frac{\sqrt{a\rho}}{(1+\rho)}$$

$$CL = \left(\frac{\lambda}{\mu+\lambda}\right)a \qquad (14)$$

$$= \frac{\rho a}{(1+\rho)}$$

$$LCL = \left(\frac{\lambda}{\mu+\lambda}\right)a - A\sqrt{a\left(\frac{\lambda}{\mu+\lambda}\right)\left(\frac{\mu}{\mu+\lambda}\right)}$$

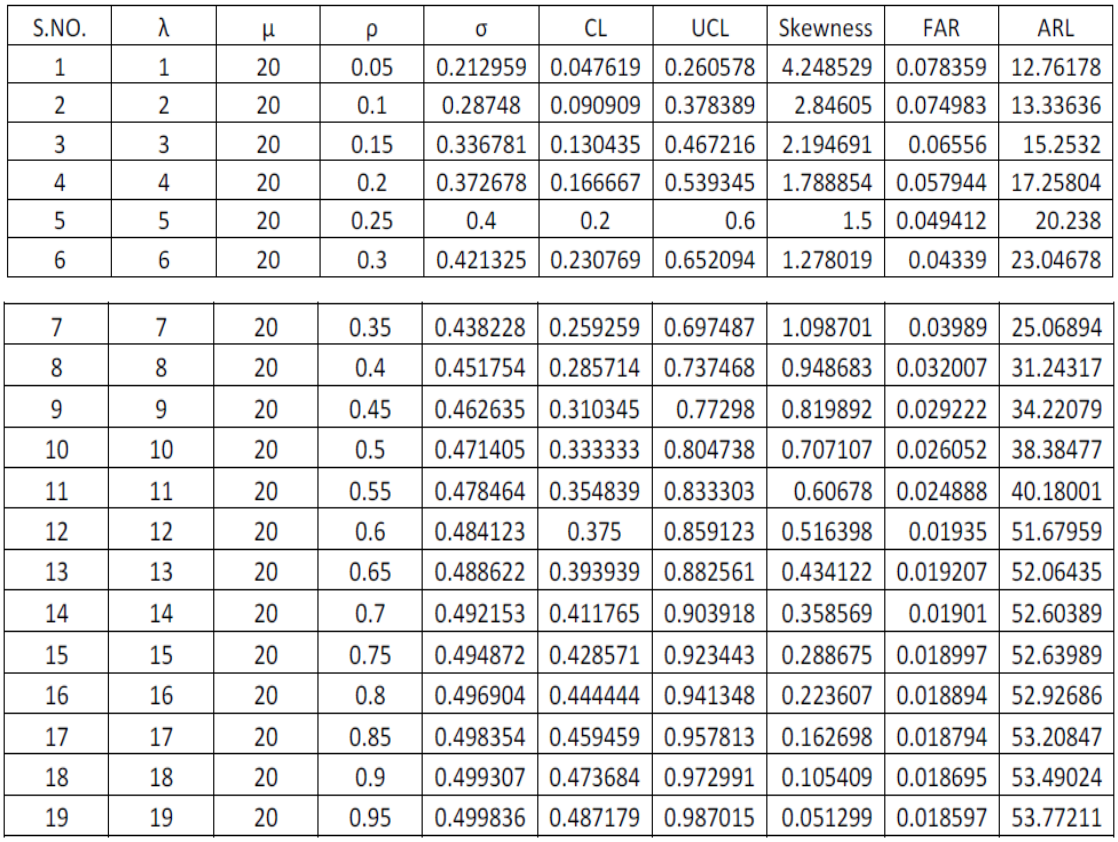
$$= \frac{\rho}{(1+\rho)} - A\frac{\sqrt{a\rho}}{(1+\rho)} \qquad (16)$$

where, traffic intensity.

V. COMPARISON OF CONTROL CHARTS

Table 1.

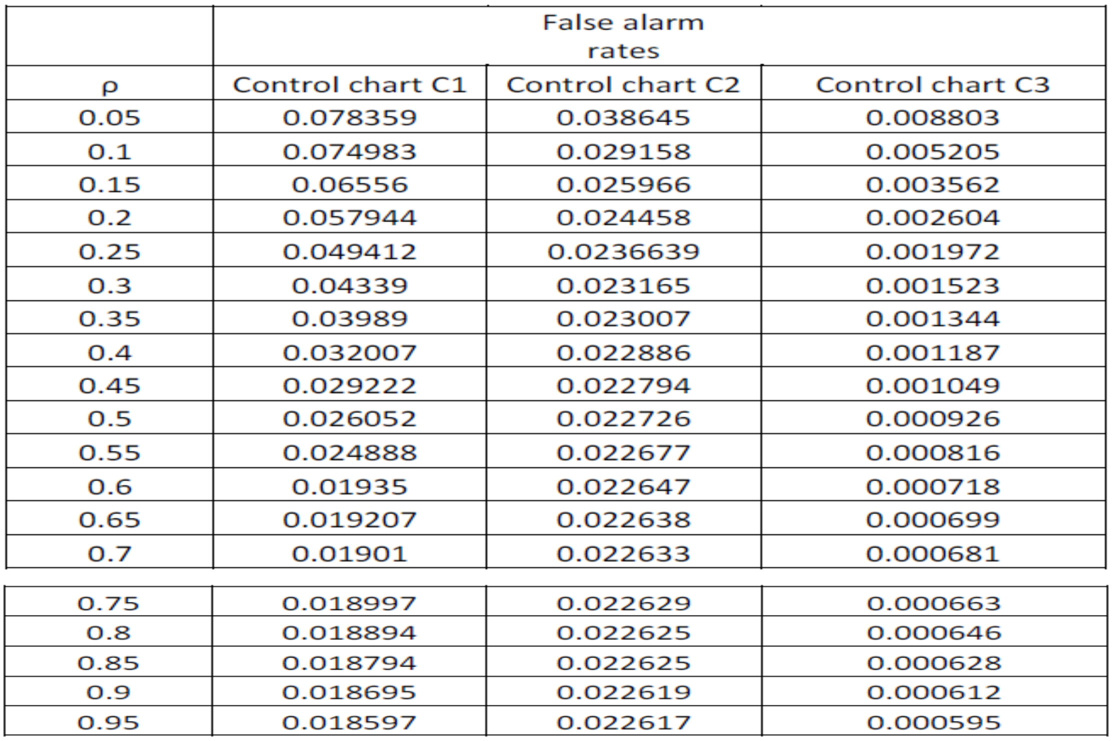
Mean, standard deviation, Upper control limit for the random variable for different values of ρ using Shewhart control charts C_1 with A=1, a=1 with L=3



S.NO.	λ	μ	ρ	σ	CL	UCL	Skewness	FAR	ARL
1	1	20	0.05	0.212959	0.047619	0.260578	4.248529	0.078359	12.76178
2	2	20	0.1	0.28748	0.090909	0.378389	2.84605	0.074983	13.33636
3	3	20	0.15	0.336781	0.130435	0.467216	2.194691	0.06556	15.2532
4	4	20	0.2	0.372678	0.166667	0.539345	1.788854	0.057944	17.25804
5	5	20	0.25	0.4	0.2	0.6	1.5	0.049412	20.238
6	6	20	0.3	0.421325	0.230769	0.652094	1.278019	0.04339	23.04678
7	7	20	0.35	0.438228	0.259259	0.697487	1.098701	0.03989	25.06894
8	8	20	0.4	0.451754	0.285714	0.737468	0.948683	0.032007	31.24317
9	9	20	0.45	0.462635	0.310345	0.77298	0.819892	0.029222	34.22079
10	10	20	0.5	0.471405	0.333333	0.804738	0.707107	0.026052	38.38477
11	11	20	0.55	0.478464	0.354839	0.833303	0.60678	0.024888	40.18001
12	12	20	0.6	0.484123	0.375	0.859123	0.516398	0.01935	51.67959
13	13	20	0.65	0.488622	0.393939	0.882561	0.434122	0.019207	52.06435
14	14	20	0.7	0.492153	0.411765	0.903918	0.358569	0.01901	52.60389
15	15	20	0.75	0.494872	0.428571	0.923443	0.288675	0.018997	52.63989
16	16	20	0.8	0.496904	0.444444	0.941348	0.223607	0.018894	52.92686
17	17	20	0.85	0.498354	0.459459	0.957813	0.162698	0.018794	53.20847
18	18	20	0.9	0.499307	0.473684	0.972991	0.105409	0.018695	53.49024
19	19	20	0.95	0.499836	0.487179	0.987015	0.051299	0.018597	53.77211

TABLE 2

Comparison of performance measure FAR for different values of ρ for control charts C_1, C_2 and C_3 with L=3

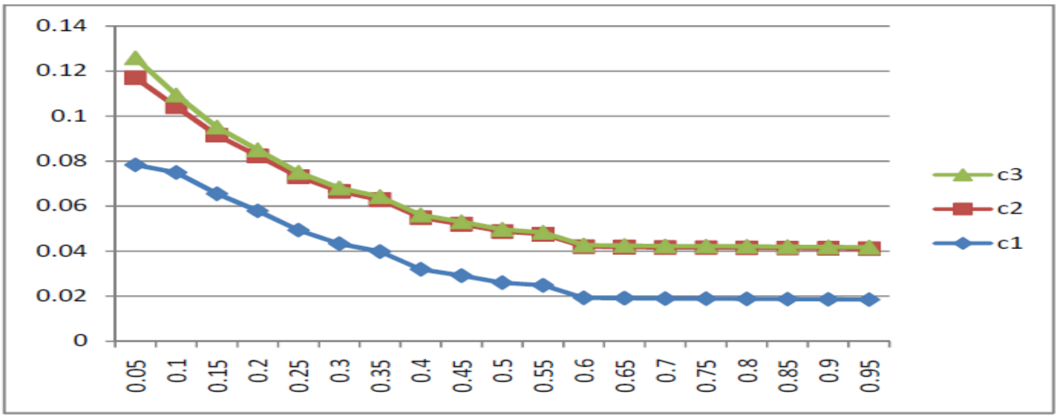


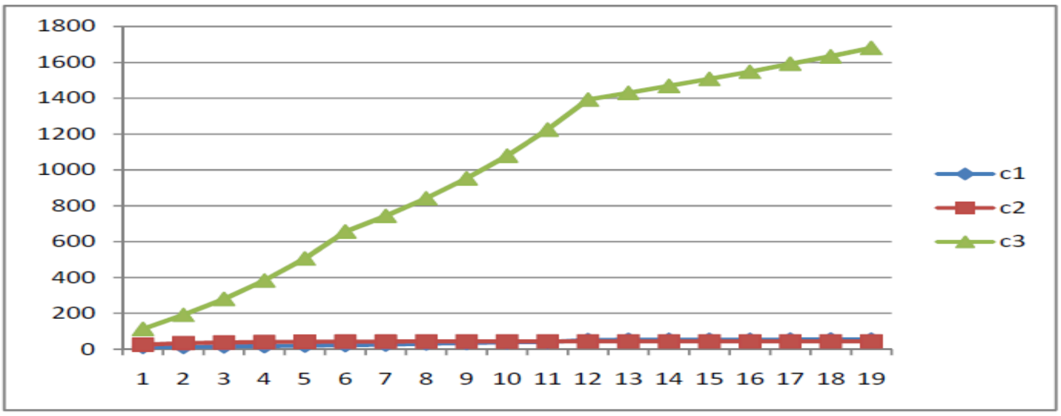
	False alarm rates		
ρ	Control chart C1	Control chart C2	Control chart C3
0.05	0.078359	0.038645	0.008803
0.1	0.074983	0.029158	0.005205
0.15	0.06556	0.025966	0.003562
0.2	0.057944	0.024458	0.002604
0.25	0.049412	0.0236639	0.001972
0.3	0.04339	0.023165	0.001523
0.35	0.03989	0.023007	0.001344
0.4	0.032007	0.022886	0.001187
0.45	0.029222	0.022794	0.001049
0.5	0.026052	0.022726	0.000926
0.55	0.024888	0.022677	0.000816
0.6	0.01935	0.022647	0.000718
0.65	0.019207	0.022638	0.000699
0.7	0.01901	0.022633	0.000681
0.75	0.018997	0.022629	0.000663
0.8	0.018894	0.022625	0.000646
0.85	0.018794	0.022625	0.000628
0.9	0.018695	0.022619	0.000612
0.95	0.018597	0.022617	0.000595

TABLE 3:

Comparison of performance measure ARL for different values of ρ for control charts C_1,C_2 and C_3 with L=3

	ARL		
ρ	Control chart C1	Control chart C2	Control chart C3
0.05	12.76177593	25.87656877	113.5976372
0.1	13.33635624	34.29590507	192.1229587
0.15	15.25320317	38.51190018	280.7411567
0.2	17.25804225	40.88641753	384.0245776
0.25	20.23799887	42.2584612	507.0993915
0.3	23.04678497	43.16857328	656.5988181
0.35	25.06893958	43.46503238	744.047619
0.4	31.24316556	43.69483527	842.4599832
0.45	34.22079255	43.87119417	953.2888465
0.5	38.38476892	44.00246414	1079.913607
0.55	40.18000643	44.09754377	1225.490196
0.6	51.67958656	44.15595885	1392.75766
0.65	52.06435154	44.17351356	1430.615165
0.7	52.60389269	44.18327221	1468.428781
0.75	52.63989051	44.19108224	1508.295626
0.8	52.92685509	44.19889503	1547.987616
0.85	53.20847079	44.19889503	1592.356688
0.9	53.49023803	44.21061939	1633.986928
0.95	53.77211378	44.21452889	1680.672269





VI. CONCLUSION

(i) Table 1 shows as ρ increases, the false alarm rate decreases and values of control limit and ARL increases.
(ii) It can be observed from Tables 2 and 3 that the value of UCL for control chart C_3 is very high as compared to control chart C_1 and C_2. This is because skewness of the underlying distribution of N is taken into consideration while obtaining control limits.
(iii) Comparing control charts C_1, C_2 and C_3, it is observed that in all three control charts as ρ increases false alarm rates decrease.

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