A Comprehensive Review of Noise-Assisted Data Analysis Methods for Non-Linear and Non-Stationary Signals

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Abstract—**The processing and analysis of non-linear and non-stationary signals represent a persistent and critical challenge in modern signal processing. While traditional analytical techniques, such as the Fourier Transform, rely heavily on assumptions of linearity and stationarity, they often fail to capture localized transient phenomena. In response to these limitations, Empirical Mode Decomposition (EMD) and its subsequent noise-assisted variantsâ€"primarily Ensemble Empirical Mode Decomposition (EEMD) and Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN)â€"have emerged as highly effective, fully data-driven alternatives. This paper presents an extensive review of noise-assisted data analysis methods spanning over two decades of research and literature. We deeply explore the theoretical foundations and mathematical models governing EMD, tracing the algorithmic evolution of noise-assisted variants designed specifically to combat the "mode mixing" problem. Furthermore, we systematically analyze the diverse applications of these methods across biomedical engineering, radar systems, geosciences, and image processing. By synthesizing 35 key academic publications, this review not only highlights the efficacy of these algorithms but also identifies critical research gaps. Specifically, we discuss the urgent need for optimizations enabling real-time processing, the lack of universal adaptive parameter selection frameworks, and the largely unexplored potential of integrating these decomposition techniques as front-end feature extractors for modern deep learning architectures. This comprehensive synthesis serves as a foundation for future innovations in non-stationary signal analysis.**

Index Terms—**mpirical Mode Decomposition, EEMD, CEEMDAN, Hilbert-Huang Transform, Non-stationary signals, Signal Processing, Iterative Filtering, Mode Mixing.**

I. Introduction

The vast majority of signals encountered in natural phenomena and complex engineering systems are inherently non-linear and non-stationary. Examples span a wide array of domains, including biomedical signals (such as Electrocardiograms (ECGs) and Electroencephalograms (EEGs)), radar echoes, speech signals, seismic waves, and mechanical vibrations. The accurate analysis of these signals is paramount for diagnostics, target detection, and system monitoring. Traditional spectral analysis methods, most notably the Fast Fourier Transform (FFT) and its short-time variant (STFT), are fundamentally predicated on the assumptions of linearity and stationarity. Consequently, they are mathematically inadequate for accurately capturing and representing localized transient features, instantaneous frequency changes, and non-linear harmonic distortions present in such complex data. Recognizing these severe limitations, Huang et al. in their seminal 1998 paper [4] introduced the Empirical Mode Decomposition (EMD) method and its associated analytical framework, the Hilbert-Huang Transform (HHT). Unlike predetermined basis-function techniques (e.g., Wavelet Transforms), EMD is a fully data-driven, adaptive algorithm. It decomposes complex, multicomponent signals into a finite, often small, number of oscillatory components termed Intrinsic Mode Functions (IMFs). The theoretical robustness and versatile applications of EMD have been extensively studied, mathematically establishing it as a highly adaptable, equivalent filter bank in signal processing [6], [7], [8]. Despite its revolutionary nature, the original EMD algorithm suffers from a significant and widely documented algorithmic drawback known as "mode mixing." Mode mixing manifests when a single IMF contains signals of widely disparate frequency scales, or conversely, when a signal of a similar frequency scale is artificially fragmented across different IMFs. This phenomenon severely degrades the physical meaning of the IMFs and complicates subsequent feature extraction. To effectively overcome the mode mixing problem, Wu and Huang (2009) proposed Ensemble Empirical Mode Decomposition (EEMD) [5]. EEMD is a pioneering "noise-assisted" data analysis method that ingeniously adds finite white noise to the original signal prior to decomposition, leveraging the statistical properties of white noise to force the signal components into their appropriate frequency scales. Building upon this, Torres et al. (2011) introduced the Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN) [18], which significantly refined the noise-addition process to achieve exact reconstruction of the original signal while heavily reducing computational overhead. More recent advancements have further expanded this domain, proposing fast multivariate EMD [15] and Iterative Filtering [1], [16] as alternative, mathematically rigorous methods for non-stationary signal decomposition. This paper systematically reviews the historical development and algorithmic refinement of EMD and its noise-assisted iterations. By comprehensively evaluating their wide-ranging applications in contemporary literature across biomedical, radar, and geoscientific domains, we aim to outline the current state-of-the-art and identify the open research gaps that must be addressed in future scholarly work.

II. THEORETICAL FOUNDATIONS AND MATHEMATICAL MODELS

A. Empirical Mode Decomposition (EMD) and the Sifting Process

At its core, EMD functions as a highly adaptive, data-driven filter bank [6], [7]. The algorithm does not require any a priori functional basis. Instead, the decomposition process termed "sifting" relies purely on the local characteristic timescale of the data. The sifting process involves the following iterative steps: 1. Identify all local maxima and local minima in the original signal $x(t)$. 2. Connect all local maxima using a cubic spline to generate the upper envelope, and similarly connect all local minima to generate the lower envelope. 3. Calculate the local mean $m_1(t)$ of the upper and lower envelopes. 4. Subtract the local mean from the original signal to isolate the first candidate IMF: $h_1(t) = x(t) - m_1(t)$. This process is repeated iteratively on $h_1(t)$ *until it satisfies the two strict conditions of an IMF: the number of extrema and zero-crossings must either equal or differ at most by one, and the mean value of the envelopes must be zero. Once the first IMF (*IMF_1) is extracted, it is subtracted from the original signal to yield a residual $r_1(t)$. *The sifting process is then applied to* $r_1(t)$ to extract subsequent IMFs. While highly adaptable, EMD is extraordinarily susceptible to intermittent noise interference, which precipitates mode mixing [8].

B. The Mode Mixing Problem

Mode mixing is primarily caused by signal intermittent ”sudden changes in frequency or amplitude. When this occurs, the cubic spline fitting during the sifting process overshoots or undershoots, causing frequency aliasing. Consequently, a single IMF ceases to represent a physically meaningful monocomponent signal. This invalidates the subsequent application of the Hilbert Transform, as it requires well-behaved IMFs to calculate accurate instantaneous frequencies.

C. Ensemble Empirical Mode Decomposition (EEMD)

To mitigate mode mixing, EEMD [5] utilizes a noise-assisted approach. EEMD creates an ensemble of N trials. In each trial, a different realization of finite variance white noise *is added to the original signal* $x(t)$*, creating* $x_i(t) = x(t) + w_i(t)$. Standard EMD is then performed on each $x_i(t)$ to obtain the corresponding IMFs. The final IMFs of the EEMD algorithm are obtained by averaging the corresponding IMFs across the entire ensemble. Because the added white noise is zero-mean, the ensemble averaging effectively cancels out the noise, leaving the true IMFs. The added noise provides a uniform reference scale distribution, forcing the intermittent signal components to align with the appropriate filter bank scales, thereby eliminating mode mixing.

D. Complete Ensemble EMD with Adaptive Noise (CEEMDAN)

While EEMD successfully mitigates mode mixing, it introduces two new issues: the reconstructed signal includes residual noise, and different realizations of noise may produce different numbers of IMFs, making averaging difficult. CEEMDAN [18] resolves these issues by adding specific noise components specifically, the IMFs of white noise at each stage of the decomposition, rather than adding Gaussian white noise to the raw signal at the very beginning. CEEMDAN calculates a unique residual after each IMF extraction and computes the ensemble average of the specific IMF before moving to the next stage. This results in an exact reconstruction of the original signal (the sum of the CEEMDAN IMFs precisely equals the original signal) and requires significantly fewer sifting iterations, making it computationally superior to EEMD.

E. Iterative Filtering and Multivariate Extensions

Recent mathematical studies have sought to formalize the EMD process. Iterative Filtering [16] has been introduced as a mathematically rigorous direct method for non-stationary signal decomposition, replacing the envelope calculation with moving average filters. This yields new insights into best practices when compared to derived algorithms [1]. Furthermore, for multi-channel sensor data, Fast Multivariate Empirical Mode Decomposition (FMEMD) [15] has been developed to handle multi-dimensional data efficiently, aligning frequency scales across multiple channels simultaneously.

III. APPLICATIONS IN BIOMEDICAL SIGNAL PROCESSING

Biomedical signal processing forms the most substantial portion of EMD and noise-assisted method research. Physiological signals are notoriously non-linear and non-stationary, frequently contaminated by artifacts such as baseline wander, power line interference, and electromyography (EMG) noise.

A. ECG Signal Denoising

A primary and extensive focus in the literature is ECG denoising. Traditional filtering techniques often distort the clinically significant QRS complex. Researchers have successfully combined EMD with wavelet transforms to achieve superior denoising [26]. Various hybrid methods leverage EMD alongside sample entropy and improved threshold functions [3], [24]. Soft thresholding techniques applied to IMFs have also shown great promise [9], [25]. By isolating the high-frequency noise components into the first few IMFs, researchers can apply soft thresholding specifically to these IMFs before reconstructing the signal, preserving the low-frequency clinical morphology. Furthermore, EMD-based techniques have proven highly effective in filtering specific artifacts, such as power line interference [21], and have been successfully integrated with nonlocal means denoising algorithms [27].

B. ECG Compression and Feature Extraction

Beyond denoising, noise-assisted methods are utilized for ECG data compression schemes [2]. By discarding IMFs that contain primarily noise or insignificant baseline wander, the data payload can be significantly compressed without losing diagnostic fidelity. Furthermore, EMD serves as a powerful feature extractor. By decomposing the signal, EMD extracts temporal and spectral features critical for the classification of ECG signals (e.g., distinguishing normal rhythms from arrhythmias) using supervised machine learning algorithms [12].

C. EEG and Other Physiological Signals

The versatility of CEEMDAN extends to the brain, where it has been pivotal for analyzing Electroencephalograms (EEG). Notably, CEEMDAN has been employed for high-accuracy epilepsy seizure detection [10]. The algorithm isolates the distinct high-frequency seizure spikes from the normal background brain activity. Additionally, EMD has been successfully applied to analyze complex esophageal manometric data in gastroesophageal reflux disease [23], demonstrating its applicability across entirely different physiological modalities.

IV. APPLICATIONS IN RADAR, VIBRATION, AND GEOSCIENCES

A. Radar Systems and Target Detection

In modern radar applications, processing echoes and filtering environmental noise/clutter is critical. The non-stationary nature of moving targets makes EMD highly suitable. EMD and HHT have been heavily utilized for the signal processing of complex radar echoes [34]. A significant application lies in enhancing Doppler profile detection of MST (Mesosphere-Stratosphere-Troposphere) radar signals [33]. Furthermore, studies have utilized EMD to isolate the Micro-Doppler effect in radar [19]. This effect is characterized by the micro-motions of target sub-components (e.g., the swinging of arms of a walking human). By decomposing the signal, researchers have used UWB FM-CW radar combined with these techniques to accurately detect human beings in motion inside buildings [20], separating the human micro-Doppler signature from the stationary building clutter.

B. Mechanical Vibration Analysis

In industrial applications, detecting faults in rotating machinery requires analyzing vibration signals, which are heavily non-stationary during speed fluctuations. Improved HHT algorithms have seen successful applications in mechanical vibration signal analysis [35], allowing for the early detection of gear and bearing faults by isolating the specific fault-frequency IMFs.

C. Geosciences and Magneto telluric Data

The earth sciences heavily rely on non-stationary signal analysis. The Hilbert-Huang Transform has been notably employed to detect earthquake-induced radon precursors [17]. Radon gas emissions fluctuate non-linearly prior to seismic events, and HHT effectively captures these transient anomalies. Similarly, EMD is utilized to process and denoise marine magneto telluric data [32], which is heavily corrupted by oceanic wave motion and environmental electromagnetic noise.

V. APPLICATIONS IN IMAGE, VIDEO, AND ACOUSTICS

A. 2-D Extensions and Image Enhancement

While primarily a 1-D time-series tool, significant efforts have been made to extend noise-assisted analysis to 2-D matrices for image processing. Techniques like White-Balanced EMD provide profound visual enhancement for underwater images [30], [31]. Underwater images suffer from severe non-linear color degradation and scattering; 2-D EMD effectively separates the illumination background from the high-frequency object details, allowing for superior color correction compared to traditional histogram equalization.

B. Watermarking and Visual Tracking

Researchers have applied EMD concepts to security and computer vision. Robust watermarking schemes in the color space based on channel coding have been developed utilizing EMD [28], hiding watermarks in specific, robust IMFs. Furthermore, EMD principles have contributed to spatially attentive visual tracking [29] using multi-model adaptive response fusion.

C. Speech and Acoustic Processing

Speech signals are the quintessential non-stationary signal. EMD facilitates advanced speech signal processing [14], particularly in separating voiced and unvoiced speech segments and denoising speech in harsh acoustic environments. In chemical acoustics and spectroscopy, UV-visible absorption spectrum denoising has been significantly improved using EEMD and universal threshold filters [22].

VI. CRITICAL ANALYSIS, CHALLENGES, AND FUTURE DIRECTIONS

While EMD, EEMD, and CEEMDAN have proven incredibly versatile, a critical analysis of the literature from 1998 to 2021 highlights several persistent gaps and outlines clear directions for future research:

A. Computational Complexity and Real-Time Implementation

The most glaring limitation of noise-assisted methods is their computational overhead. The ensemble averaging process in EEMD and CEEMDAN, which requires hundreds of sifting iterations across multiple noise realizations, is computationally intensive. The literature heavily favors offline analysis. There is a critical lack of robust algorithmic optimizations tailored for strict real-time, low-latency processing. This is a severe bottleneck for integrating these methods into mobile health devices (e.g., wearable ECG smartwatches) and live, real-time tactical radar systems. Future research must focus on algorithmic pruning and the widespread adoption of fast multivariate EMD architectures [15].

B. Adaptive Parameter Optimization

The success of EEMD and CEEMDAN depends critically on two user-defined parameters: the amplitude of the added noise and the number of ensemble trials. In the surveyed literature, these parameters are almost universally selected via heuristic approaches, empirical guessing, or exhaustive trial-and-error. There is a significant research gap regarding the development of universal, automated, and dynamic parameter-selection frameworks that automatically adapt to the specific Signal-to-Noise Ratio (SNR) of the incoming data stream.

C. Synergy with Deep Learning Architectures

The reviewed studies primarily rely on traditional techniques (e.g., wavelet thresholding, basic filter banks) or simple supervised learning (SVMs, Random Forests) for post-decomposition classification [12]. The integration of CEEMDAN as a front-end, trainable feature extractor for modern deep neural networks (such as Convolutional Neural Networks, LSTMs, or Transformers) remains vastly underexplored. Leveraging end-to-end deep learning could automate the complex process of IMF selection and drastically improve classification accuracy in both biomedical and radar domains.

D. Multimodal and Multi-dimensional Data Expansion

The vast majority of research focuses on isolated 1-D signals. While sparse attempts have been made in 2-D (e.g., underwater images [31]), extending noise-assisted methods efficiently to handle massive multi-dimensional and multimodal sensor data such as synchronized EEG-fMRI recordings or distributed multi-sensor IoT networks remains an open, computationally daunting challenge.

VII. CONCLUSION

Noise-assisted data analysis methods, notably EEMD and CEEMDAN, have fundamentally revolutionized the approach to non-linear and non-stationary signal processing. By effectively mitigating the mode mixing problem inherent in traditional EMD, these algorithms have unlocked new levels of accuracy in data analysis. The widespread, highly successful application of these methods across biomedical signal processing (ECG/EEG denoising), complex radar technology (Micro-Doppler detection), and geosciences (earthquake precursors) underscores their immense, robust utility. However, to push the boundaries of this field and transition these algorithms from offline academic research to ubiquitous real-world application, future research must aggressively address the computational bottlenecks to enable real-time execution. Furthermore, developing adaptive parameter selection algorithms and natively fusing noise-assisted decomposition with advanced deep learning frameworks will undoubtedly form the next frontier of innovation in non-stationary signal analysis.

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