SEMI-MARKOV COMPARISON OF CORRECTIVE AND PREVENTIVE MAINTENANCE IN CEMENT GRINDING: A SIMULATION-BASED RELIABILITY AND PROFIT STUDY

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***Abstract*—Cement grinding relies on interacting electrical and mechanical equipment, for which an apparently minor fault can reduce throughput or progress to a complete interruption. This study compares two maintenance policies within a common seven-state stochastic reliability model. Model I permits a developing or neglected fault to progress to serious failure, Model II replaces that route with successful preventive intervention. The models retain identical category-specific fault occurrence intensities and use semi-Markov transition distributions, regenerative first-passage equations, and renewal-reward measures. Numerical inputs originate from a observed 24-month cement-grinding dataset containing 214 fault events over 16,259.13 operating hours, unobserved escalation and restoration behaviour is specified as hypothetical. Under the exponential particular case, operating-time mean time to system failure increases from 164.10 h (corrective policy) to 207.17 h (preventive policy), a 26.24% increase. Long-run binary availability increases from 97.88% to 98.32%, while modelled profit rises from approximately ₹89,647 to ₹90,430 per hour under the stated throughput and cost assumptions. Reliability functions and a break-even policy-cost calculation provide additional insight into this trade-off. These are conditional simulation-based findings, not measured improvements at an industrial installation. The results illustrate a transparent framework for evaluating when early intervention in developing faults justifies its maintenance burden.**

***Index Terms*—cement grinding, corrective maintenance, preventive maintenance, regenerative point technique, semi-Markov process, reliability, expected profit**

I. Introduction

Reliable cement grinding is necessary because material feeding, mill drives, bearings, gearboxes, conveyors, separators, fans and lubrication devices function as an interconnected production system. A minor motor disturbance or bearing vibration may initially reduce throughput rather than stop the line. A major electrical, mechanical or control failure can instead interrupt production until the affected equipment is restored. Maintenance decisions therefore affect both the frequency of unplanned outages and the quantity of cement produced between outages. In particular, a policy that postpones attention to early warning symptoms can create a failure pathway that would not exist if intervention occurred promptly.

Conventional reliability theory distinguishes failure-free service, maintainability and system availability (Barlow & Proschan, 1965). Optimal preventive maintenance can be formulated as a renewal decision in which the additional intervention burden is balanced against averted failures (Barlow & Hunter, 1960). Later surveys describe a broad range of periodic, age-based and condition-based policies (Wang, 2002; de Jonge & Scarf, 2020). Condition monitoring has made it increasingly practical to base interventions on evidence of deteriorating equipment condition rather than calendar age alone (Jardine et al., 2006; Nunes et al., 2023).

The cement industry presents a useful application setting because production contribution may be substantial even when an individual maintenance event is relatively inexpensive. Integrated cement-process optimization must account for production, maintenance and energy-related constraints rather than evaluating components in isolation (Swanepoel et al., 2014). Cement machinery monitoring studies also identify the importance of detecting faults in rotating equipment before severe degradation occurs (Rusiński et al., 2013).

This paper addresses one narrowly defined question: holding the initial six-category fault intensities fixed, what changes when developing faults are restored through preventive maintenance rather than allowed to progress to complete failure? It uses two explicitly comparable stochastic models rather than asserting that preventive maintenance is always preferable. The contribution is an auditable first-passage and regenerative reward comparison in which the fault counts, progression assumptions, partial-production rewards, corrective costs and additional preventive costs remain distinguishable. A companion thesis investigation of PLC inspection and lubrication redundancy addresses a separate decision and is not analysed here.

II. Previous Work and Study Contribution

Reliability models have long used Markov and semi-Markov processes to represent repairable engineering systems. The semi-Markov approach allows arbitrary state-holding distributions, while the exponential special case provides an interpretable computational benchmark (Limnios & Oprişan, 2001; Janssen & Manca, 2006). Nakagawa and Osaki (1975) showed how standby and repair operations alter stochastic system behaviour, including first failure and repairman activity. Maintenance-optimization studies emphasize that minimum failure probability and maximum profit need not be achieved by the same policy (Dekker, 1996; de Jonge & Scarf, 2020).

These studies motivate a structured comparison rather than a claim of new regenerative theory. The specific contribution of this paper is the coupling of (a) a six-category cement-grinding fault register, (b) a separate neglected-fault progression mechanism, (c) production at reduced capacity, and (d) a renewal-reward profit measure on the **same** system boundary. The modelling choices deliberately prevent a second assumed failure-rate reduction from being applied on top of a preventive-restoration pathway. Consequently, the

numerical differences can be attributed to the declared policy transition and its costs rather than simultaneous, untraceable modifications of the input data.

III. Materials and Methods

3.1 Data source, exposure and categories

The numerical case draws on an observed operational workbook prepared for the cement-grinding reliability thesis. It describes 729 daily records spanning 1 June 2024 to 30 May 2026; the modeled aggregate operating exposure is 16,259.13 hours. The workbook contains 214 fault events, 214 corrective work orders, 121 preventive maintenance activities and separate PLC alarm records. It was **not** authenticated against industrial plant logs. No on-site experiment, fault intervention trial or machine-level diagnostic validation is claimed. The time interval identifies the observed observation window, not a verified historical collection programme.

The six failure modes have a common operating-time exposure. The category-specific occurrence-rate estimator is

$$\hat{\lambda}_i = \frac{N_i}{T_{op}}, \qquad T_{op} = 16\,259.13\ h, \qquad i = 1, \ldots, 6. \qquad (1)$$

Table 1 gives the category frequencies. The event register does not contain dependable onset times for every inter-failure interval; exponential fault-arrival assumptions therefore define a numerical *particular case* rather than constituting a tested law. Interruption severity is also separate from fault category: the state model groups events by expected operational consequences and does not attempt to reconstruct every one of the 109 observed full stoppages.

Table 1. Observed fault classification and rate estimates.

Category	Typical operational interpretation	Events	Rate (h^{-1})	Share (%)
Minor electrical	Reduced-capacity disturbance	49	0.003014	22.90
Major electrical	Full production interruption	31	0.001907	14.49
Minor mechanical	Reduced-capacity disturbance	71	0.004367	33.18
Major mechanical	Full production interruption	33	0.002030	15.42
Developing / neglected	Warning or deterioration condition	22	0.001353	10.28
PLC / instrumentation	Serious control disturbance in model	8	0.000492	3.74
Total		214	0.013162	100.00

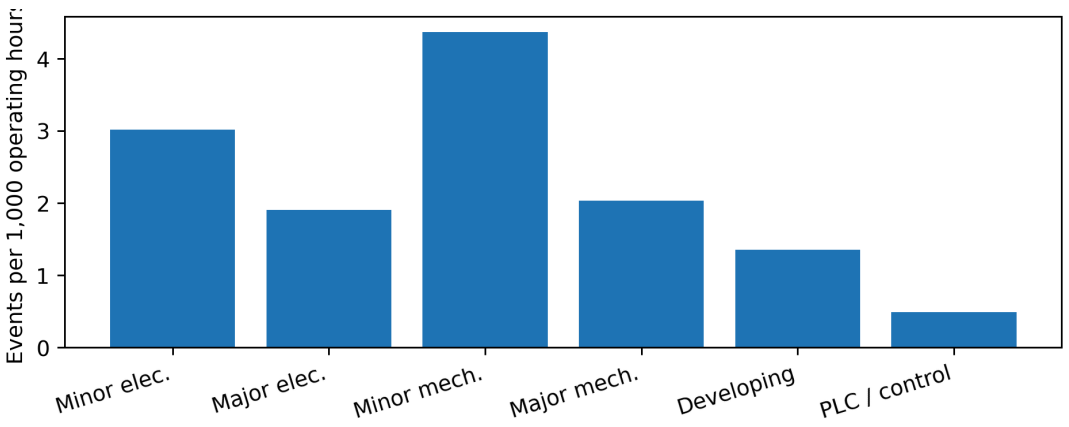

Figure 1. Estimated fault occurrence intensities by category using the observed study exposure.

3.2 State space and maintenance policies

Both policies employ states S_0 (fully operative), S_1 (minor electrical fault, partial operation), S_2 (major electrical failure), S_3 (minor mechanical fault, partial operation), S_4 (major mechanical failure), S_5 (developing or neglected fault) and S_6 (complete PLC/instrumentation failure). Initially the system occupies S_0. The complete-failure states are S_2, S_4 and S_6; S_1, S_3 and S_5 retain a declared reduced production factor. Repair completion returns the system to S_0 and is treated as restoration to an as-good-as-new state.

In Model I, S_5 ultimately progresses to S_2 or S_4, with an assumed electrical escalation proportion $a = 31/(31 + 33)$. In Model II, S_5 returns to S_0 after preventive maintenance. Both policies permit competing online repair and progression from the minor-fault states; their rates are unchanged across models. One maintenance facility is assumed. Simultaneous component failures, maintenance queues, common-cause effects and component age accumulation are outside the seven-state approximation.

Table 2. Shared parameters and the sole policy-specific transition.

Element	Model I: corrective	Model II: preventive
Initial fault intensities	Six rates from Table 1	Same six rates
Minor electrical restoration	Mean 0.7412 h; progression rate 0.080 h^{-1}	Identical
Minor mechanical restoration	Mean 0.8451 h; progression rate 0.075 h^{-1}	Identical
Major electrical restoration	Mean 3.4790 h	Identical
Major mechanical restoration	Mean 3.8042 h	Identical
PLC/control restoration	Mean 2.4875 h	Identical
State destination	or after mean 10 h	after mean 0.50438 h
Additional preventive cost	None	₹2,506.85 per intervention plus ₹850/day hypothetical overhead

Reduced-capacity multiplier	0.65	0.65

The progression intensities in Table 2 and the assumption that successful preventive maintenance always averts escalation are hypothetical. Although the preventive workbook contains 121 activities, the state residence time of 0.50438 hours is a scenario approximation, not a directly measured online-maintenance duration.

3.3 Semi-Markov transition distributions

Let $Q_{ij}(t)$ be the probability of transition from state i to state j within elapsed time t after entry to i. For the initial operative state with total competing fault rate $\Lambda = \sum_i \lambda_i$, the exponential special case gives

$$Q_{0i}(t) = \frac{\lambda_i}{\Lambda}\left(1 - e^{-\Lambda t}\right), \quad p_{0i} = \frac{\lambda_i}{\Lambda}. \qquad (2)$$

Minor electrical repair at rate μ_1 competes with escalation at rate θ_1; minor mechanical repair at μ_3 competes with escalation θ_3. Therefore,

$$p_{10} = \frac{\mu_1}{\mu_1+\theta_1}, \quad p_{12} = 1 - p_{10}, \quad p_{30} = \frac{\mu_3}{\mu_3+\theta_3}, \quad p_{34} = 1 - p_{30}. \qquad (3)$$

For a general repair-time distribution, transition completion is represented by the corresponding cumulative distribution function. The mean sojourn time in state i is the integrated survival probability,

$$m_i = \int_0^\infty \left[1 - \sum_j Q_{ij}(t)\right] dt. \qquad (4)$$

$$\tilde{Q}_{ij}(s) = \int_0^\infty e^{-st}\, dQ_{ij}(t).$$

Here the second expression is the Laplace–Stieltjes transition transform. Thus the analysis is a semi-Markov construction, specialized to a finite-state continuous-time Markov chain only for the numerical case. Transition probabilities from each nonabsorbing state sum to one.

3.4 Mean time to complete system failure

Let T_F be the first entry time into any complete-failure state. The mean satisfies regenerative first-step equations with S_2, S_4, S_6 made absorbing. The two closed-form expressions are

$$MTSF_I = \frac{m_0 + p_{01}m_1 + p_{03}m_3 + p_{05}m_5}{1 - p_{01}p_{10} - p_{03}p_{30}}, \qquad (5)$$

$$MTSF_{II} = \frac{m_0 + p_{01}m_1 + p_{03}m_3 + p_{05}m_5^{PM}}{1 - p_{01}p_{10} - p_{03}p_{30} - p_{05}}. \qquad (6)$$

The additional return term p_{05} in the Model II denominator is the algebraic expression of preventive restoration rather than neglected-fault escalation. $MTSF$ is the expectation of the first *complete* interruption, not the reciprocal of the overall event intensity. For a finite-state exponential particular case, equivalent matrix evaluation is possible:

$$R_k(t) = e_0^T e^{Q_{T,k}t} 1, \qquad MTSF_k = e_0^T \left(- Q_{T,k}\right)^{-1} 1. \qquad (7)$$

$Q_{T,k}$ is the transient generator after absorbing states are removed; Equation (7) also gives the entire reliability function. Both first-step and matrix calculations were used as computational checks.

3.5 Availability, maintenance workload and profit

For each model, the complete generator Q_k includes restorative transitions. The stationary state probabilities solve

$$\pi_k Q_k = 0, \quad \sum_i \pi_{i,k} = 1, \quad A_k = \pi_{0,k} + \pi_{1,k} + \pi_{3,k} + \pi_{5,k}. \qquad (8)$$

This binary availability definition counts reduced-capacity operation as available, but its economic contribution is weighted by $\eta = 0.65$. With contribution $C_0 = 130 \times ₹710 = ₹92\,300$ per full-capacity operating hour, mean expenditure per event c_{ij} and policy overhead $C_{fix,k}$, the long-run reward is

$$P_k = C_0\left[\pi_{0,k} + \eta\left(\pi_{1,k} + \pi_{3,k} + \pi_{5,k}\right)\right] - \sum_{i \neq j} \pi_{i,k} q_{ij,k} c_{ij} - C_{fix,k}. \qquad (9)$$

Event costs are attached only to the relevant maintenance transitions. The model does not impose an additional lost-revenue penalty for the same downtime already represented through the state reward. Repairman occupancy can be evaluated from probabilities of service states, but the aggregate major restoration occupancy is a workload proxy unless repairs and replacements have separate observed holding times.

IV. Results

4.1 Baseline performance and reliability

The exponential particular case was evaluated with the parameters in Tables 1 and 2. State occupancy, first-passage reliability and profit were computed from Equations (7)–(9). Table 3 contains the main results. The numerical values reproduce the revised thesis baseline within rounding. The preventive policy raises MTSF by 43.07 hours, or 26.24%, while the availability gain is about 0.45 percentage points. The production-equivalent reward and additional maintenance costs must be considered together to interpret the net profit difference.

Table 3. Baseline comparative results from independent matrix evaluation.

Metric	Model I	Model II	Difference (II − I)
MTSF (h)	164.10	207.17	+43.07
Reliability at 24 h	0.87353	0.89085	+0.01732
Reliability at 48 h	0.75416	0.79337	+0.03921
Binary availability	97.88%	98.32%	+0.45 pp
Complete downtime per 1,000 h	21.22 h	16.76 h	−4.46 h
Expected total maintenance expense	₹101.42/h	₹125.19/h	+₹23.77/h
Expected profit	₹89,647.48/h	₹90,429.96/h	+₹782.48/h

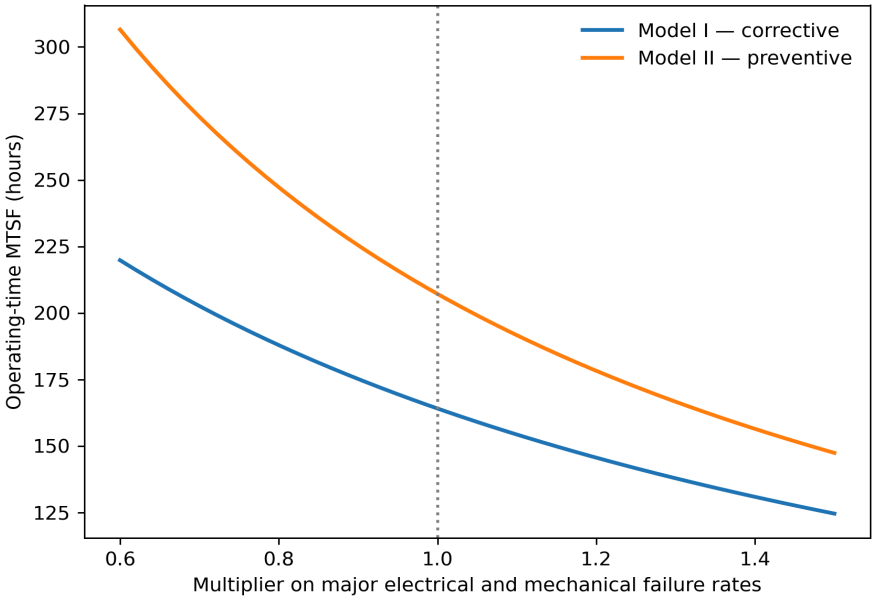

Figure 2. MTSF response to a common multiplier on the major electrical and mechanical fault rates; every plotted point is obtained from the first-passage equation.

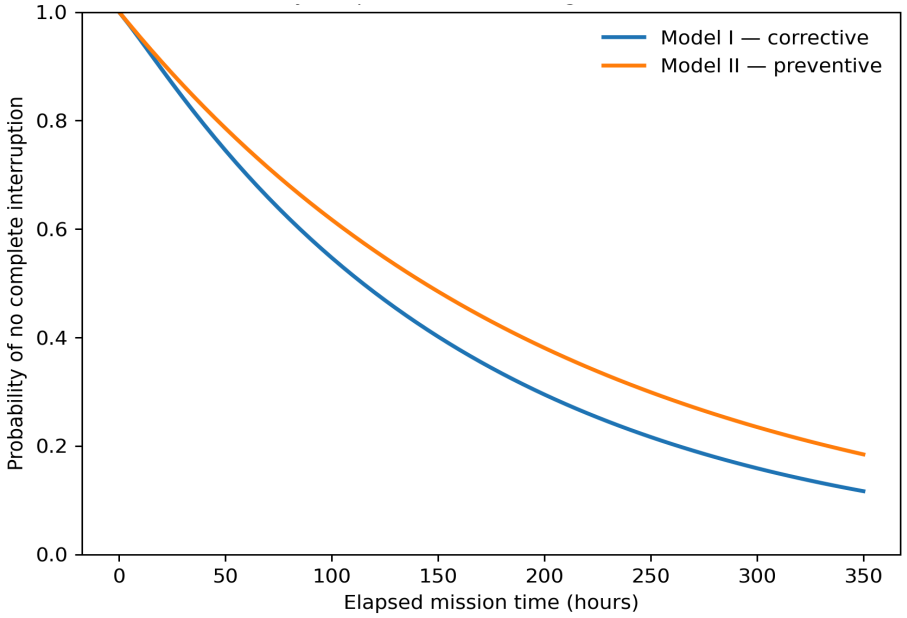

Figure 3. Model-specific probability of avoiding a complete system interruption up to the stated mission time.

Figure 2 shows that major-fault intensification reduces MTSF under both policies. Figure 3 shows that the preventive policy has higher reliability over the plotted interval under this particular parameter set. Neither finding demonstrates universal dominance: an alternative fault-progression process, PM duration or inspection effectiveness could change the ordering. The maximum plotted horizon is a sensitivity window, not a verified service-life limit.

4.2 Cost sensitivity and conditional break-even

The modelled economic gain from preventive maintenance is $\Delta P = P_{II} - P_{I} = ₹782.48$ per modelled hour. If an unbudgeted additional overhead B is imposed exclusively on the preventive policy without changing transition rates or productivity, the difference becomes

$$\Delta P(B) = 782.48 - B; \qquad B_{BE} = ₹782.48\, h^{-1}. \qquad (10)$$

Figure 4 displays this accounting break-even. It is separate from a physically optimized maintenance interval: changing actual intervention frequency will generally change the time spent in state S_5, the number of work orders and possibly fault hazards. An economically optimal schedule would need those dependencies estimated rather than assumed.

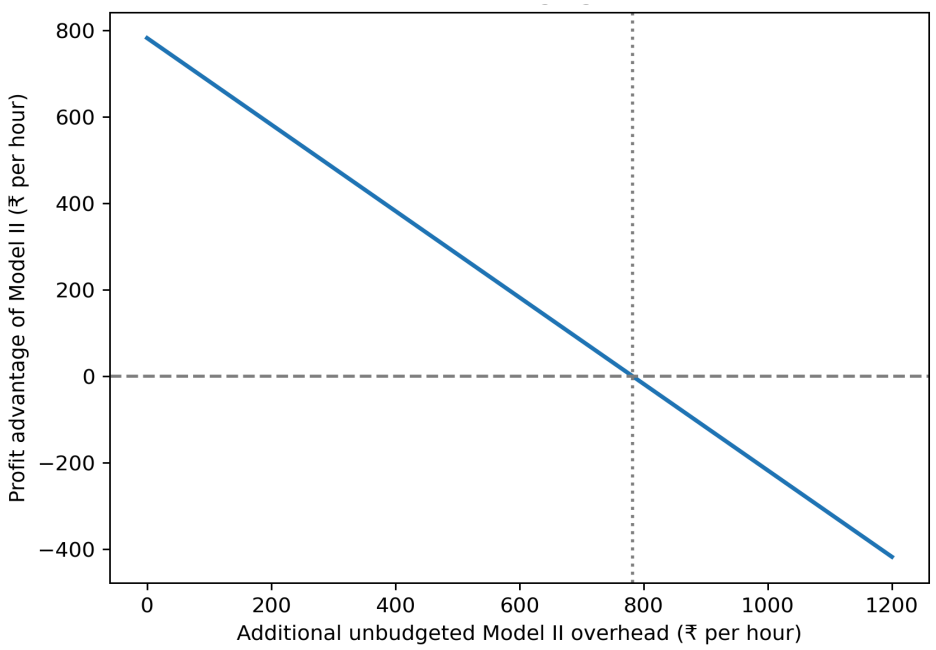

Figure 4. Incremental unbudgeted preventive-policy overhead at which the modelled profit advantage is exhausted.

Table 4. Selected conditions affecting the comparison.

Decision condition	Mathematical implication	Interpretation
More frequent major faults	Both models' first-passage times decline	Better fault prevention has greater potential value
Neglected faults promptly restored	Model II includes return	Preventive model avoids the assumed escalation path
PM requires a full shutdown	Online PM availability reward is no longer valid	Model II availability and profit must be recomputed
Additional PM overhead > ₹782.48/h	Other inputs held fixed,	Maintenance expense can outweigh the modelled gain
Incomplete escalation histories	and neglected progression remain assumed	Scenario sensitivity, not causal inference

4.3 Interpretation against the research question

In this seven-state abstraction, avoiding escalation from the developing-fault state has two consequences: it prevents a direct route to the major failure states and removes the subsequent longer restoration occupation from the renewal cycle. The increase in MTSF is therefore larger than the increase in availability. MTSF tracks first passage from a fresh start, whereas availability includes repeated restoration and partial operation. These quantities answer different managerial questions and should not be merged into a single performance index.

The economic reward adds a third perspective. The policy change increases the expected maintenance-expense rate because preventive interventions and an extra policy overhead are included, yet the modeled contribution rises enough to offset this burden. This result depends strongly on the assumed throughput margin, the partial-production factor, and the simplifying assignment of a neglected fault to either electrical or mechanical escalation. The workbook provides frequencies and work orders but does not establish a counterfactual history in which the same equipment is observed with and without PM. Accordingly, no field-level causal improvement is inferred.

V. Discussion

The principal design lesson is that condition-triggered maintenance should be assessed through explicit transitions, not a blanket reduction in every failure rate. Applying both an assumed 12% reduction and an automatic return from the developing-fault state would artificially amplify the intervention's benefit. The present models instead retain the original intensities and change only how the developing state exits. This distinction is important when maintenance engineers use historical event tables that lack detailed onset timestamps.

Similarly, a stochastic availability improvement does not establish a proportional production improvement. In a grinding line, a conveyor slowdown or a bearing warning may permit continued processing but at reduced output. The binary state reward and the capacity-weighted state reward were deliberately reported

separately. Their values can change under revised capacity factors without altering the definition of complete failure.

The analysis also highlights limits of a single-service-facility assumption. Multiple technicians may undertake overlapping inspections or online actions, while a major mill outage may depend on spare-part availability and permit-to-work delays. A more detailed semi-Markov model could include repair waiting time, spare inventories and non-exponential wear. Such improvements should follow better data capture rather than indiscriminate addition of unidentifiable model states (Lawless, 2003; Janssen & Manca, 2006).

The case supplies a computational benchmark for cement machinery rather than performance certification. It is not a controlled industrial intervention and provides no experimental confirmation that a particular PM programme can deliver 26.24% more MTSF. Verification requires timestamps, operating exposure by subsystem, consistent shutdown coding, an independently assessed preventive programme and uncertainty estimates.

VI. Conclusion

Using an observed cement-grinding dataset and a common seven-state stochastic structure, this study compared corrective maintenance with a preventive policy that interrupts the progression of neglected faults. The mathematical particular case gave MTSF values of 164.10 and 207.17 hours, availability of 97.88% and 98.32%, and hourly profit of approximately ₹89,647 and ₹90,430 for Models I and II, respectively. The larger first-passage advantage arose from eliminating the assumed neglected-fault escalation transition. Conditional accounting suggests that an additional ₹782.48/h of unbudgeted preventive overhead would eliminate the baseline profit advantage if every other input remained fixed. The main contribution is a reproducible regenerative framework for examining the reliability–cost trade-off of early maintenance decisions.

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